Standard 5.NF.5 Interpret multiplication as scaling.

a. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. For example, the products of expressions such as 5 x 3 or 1/2 x 3 can be interpreted in terms of a quantity, three, and a scaling factor, five or 1/2. Thus in addition to knowing that 5 x 3 = 15, they can also say that 5 x 3 is five times as big as three, without evaluating the product. Likewise they see 1/3 x 3 as half the size of three.

b. Explain why multiplying a given number by a fraction greater than one results in a product greater than the given number (recognizing multiplication by whole numbers greater than one as a familiar case); explain why multiplying a given number by a fraction less than one results in a product smaller than the given number; and relate the principle of fraction equivalence. For example, 6/10 = (2x3)/(2x5). In general, a/b = (n x a)/(n x b) has the effect of multiplying a/b by one.

Please Note: Apply and extend previous understandings of multiplication and division to multiply and divide fractions (Standards 5.NF.3–7).

Key Elements: Unlike multiplication with whole numbers where the product is greater than both factors, the conceptualization of multiplication as scaling can produce an instance where, when you multiply by a fraction less than one, your number is smaller than at least one of your original factors. It also means that when you multiply a fraction greater than one you get a larger product, and can explain why. The students should also be able to explain that to make an equivalent fraction you are simply multiplying by one in fractional form.

Notes: This standard is a challenge for many students because it starts to challenge the way they think of multiplication. To many students it actually seems more like division than multiplication; and in all fairness, there is some truth to it. Within fractions is the inherent need to divide the whole into equal sized pieces and then identify the number of those pieces you are dealing with.

In multiplication there are several ways that students often conceptualize, or think about, multiplication. The first one that many seem to fall back on is that of repeated addition, where a number is repeated any given number of times. For example 5 repeated 3 times, or ¼ repeated 6 times. This idea fits within the larger groups and group size conceptualization, which more versatile and more advanced. An example may be 3 groups of five, or 3 boxes where each box weighs 5 pounds.

The group size conceptualization allows for multiplication with fractions. For example, you may have a group of 12 items (group size) and ⅓ of this group is red. -(insert models4) This type is often expressed as ⅓ of a group of 12, or ⅓ of the box of 12 marbles are red. It should be noted that this does feel less like the multiplication that we normally think of, as it has a division feel. This is what we call **scaling**. Scaling is where a factor changes the other by taking the original amount and adjusting it by “breaking” it into the indicated number of equal pieces (denominator), with the numerator telling how many of those pieces of the original amount it is “looking for.”

The following examples help illustrate this point and may be a way to help students adjust their understanding of multiplication to fit this within their existing schema. It should be noted that the answers to multiplication with fractions always refer back to the original whole.

Whole number multiplied by a fraction

This type of multiplication is where you scale a whole number. Though the following situation shows the product as being a whole number that is not always the case, it is possible to have a fractional or mixed number answer.

- There were 10 people at a party. 1/5 of them were wearing blue party hats. How many people have blue party hats? -(insert model 5)
- There were 10 cups of butter being used in a recipe. ¾ of that had to be melted to be put into the frosting. How much butter goes into the frosting? -(insert model 6)

As you can see, the scaling idea becomes quite clear that you are dividing one group into equal sized pieces and you are looking at how many of those (from the original amount) you are using, thus giving you a smaller product.

Fraction Multiplied by a Fraction

In many ways this is the hardest type to understand. You scale a fraction, by a fractional amount. The following examples should help clarify this idea.

- Carol has made 6/8 of a cake. 1/3 of that cake was frosted blue. How much of the total cake is blue? -(insert model 7)
- Bree walks to school everyday. She walks a total of ¾ of a mile. Today she skipped 2/3 of the distance. How much of a mile did she skip? -(insert model 8)

In both of these instances the original factor is being scaled to an even smaller fraction, by breaking the original denominator into a multiplicatively related denominator and the numerator scaled accordingly as well. Another way of saying it: you are taking a fraction of fraction.

Fraction Multiplied by a Mixed Number

The scaling idea works very similarly to what has already been discussed, but often results in fractions larger than a whole, or at least larger than one of the original factors, which is different from multiplying a fraction with a fraction.

- There were 2 ½ cakes left over from a celebration; of those cakes, 1/3 of them had purple frosting. How much of a cake was there purple frosting?

Mixed Numbers Multiplied by Mixed Numbers

This is where students have to connect their understanding of whole number multiplication with fractions. It is also a useful place to bring up the distributive property. There are two main ways of solving these problems. First, you can turn both into fractions greater than one and multiply as shown above, often getting really large numerators. Second, you can multiply the whole numbers, the first whole number with the second fraction, then the second whole number with the first fraction, and finally the two fractions together. Add all of the partial products together to obtain the answer. Note: the area model is a great connection to this for the students.